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Given Correlation Manifolds and their Application in Blind Channel Identification

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Abstract: This paper introduces the new objects named polynomial statistics. We discuss their properties and applicability to the problem of Blind Channel Identification which is a highly topical issue at the moment. This paper proposes a solution to this problem which is based on using polynomial representations of finite random sequence. Algorithms of Blind identification that was described in this paper use a factorization property of so-called given correlation manifolds.

Key Words: Blind channel identification, polynomial statistics, zero-correlation manifold, given correlation manifold.

I. INTRODUCTION

The last few years have seen a growing interest in the so-called 'blind problem' [1-6]. In general terms, blind processing can be defined as digital processing of unknown signals which are transmitted through a linear channel or medium with unknown characteristics and additive noise. In contrast to the blind identification method, the classical method of channel identification analyses the output signal for an input signal with given characteristics. The reason why blind channel identification (BCI) has lately attracted a lot of researchers' interest is the apparently high potential that this method has for application in the rapidly developing mobile communications industry.

Blind channel estimation is applied in a variety of industries other than telecommunications. Some examples include compensation for signal distortions caused by multipath propagation effects in radiolocation and radio navigation systems; correction of linear distortion in image processing systems; seismic signal processing in geophysics; distortion compensation in speech recognition systems.

In solving the problem of BCI, identifiability of a system is an important issue. 'Blind' identifiability of a system can be defined as a possibility to restore its transfer function and/or impulse response (IR) based on the data extracted from the output signal alone, with accuracy sufficient for obtaining the correct complex factor. SISO channels identifiability conditions are the scope of statistical identification suggesting that there is a set of output signal representations which are characterized by the same impulse response of the channel. In this case the system is identifiable if its input signal is a non-stationary or non-Gaussian random process.

Y. Sato must have been the first one to take advantage of the non-Gaussian character of information signals in digital amplitude-modulation systems. In 1975 he proposed an algorithm of direct blind equalization [7]. Five years later D. N. Godard adapted Sato algorithm [8] for the case of combined amplitude-phase modulation (the algorithm proposed by Godard is also known as Constant Modulus Algorithm). Nowadays various criteria of adaptation of linear equalizers are used in many algorithms of blind channel identification and channel equalization. Such algorithms form the class of stochastic gradient algorithms, or Busgang algorithms. The main disadvantages of these algorithms include a relatively slow convergence, the need for valid entry conditions, a greater computational complexity which is due to nonlinear character of equalizer coefficients optimization procedure, and a low noise immunity.

Another class of blind identification algorithms, which were introduced only recently, includes algorithms using the rule of maximum likelihood. Such algorithms are characterized by high asymptotic efficiency and noise immunity, providing for reliable channel estimates. At the same time, those algorithms have two major problems, namely, computational complexity and local maxima [9].

One effective technique to develop blind identification algorithms is the so-called method of moments. This method basically consists in taking away the equations that link input and output signals of system and substituting for them equations linking moment functions corresponding to those signals. Although, in terms of their asymptotic convergence, the estimates obtained by the method of moments are not the best ones (see [10]), the method usually produces a fair channel estimate, even though it does not apply the nonlinear optimization procedure. It gives computational advantages in comparison with similar methods, including a likelihood approach. In addition to that, the method of moments does not require the prior knowledge of stochastic distribution for signals and noise, which, in context of 'blind problem', is a key advantage. It is well known that the covariance function of a stationary process represented by a linear system output does not provide any information on the phase of its transfer

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function. The channel can only be identified for a narrow class of systems with minimal phase. This explains the researchers' interest in high order statistics and non-Gaussian models of entry signals [9, 11]. It is possible to use second-order statistics for blind channel identification in case of a non-stationary model of input or output signals and periodically-correlated signals. Identifiability of telecommunication channels with non-stationary input was demonstrated in [12]. The method of moments usually uses cumulant spectrums for channel estimation, since equations describing an unknown channel can be written in a simple algebraic form. In this paper we introduce a new approach to the synthesis of statistical blind identification algorithms, which is based on a polynomial representation of the moments of random sequences [13].

II. POLYNOMIAL STATISTICS AND MANIFOLDS GENERATED BY THEM

Let us assume that $\mathbf{x}\in C^n$ is a complex random vector set by the probability density function $f_X\left(x_1,...,x_n\right)$, and $x(z), z\in C$ is a complex random polynomial of degree $n-1$ with vector of coefficients $\mathbf{x}\in C^n$.

A polynomial moment of order $(k+m)$, where $k=k_1+k_2+...+k_r$ and $m=m_1+m_2+...+m_r$, of random vector $\mathbf{x}\in C^n$ is a polynomial belonging to ring $C\left[z_1,...,z_r\right]$ over the field of complex numbers. It is generated as follows:

$$
\begin{aligned}
&P^x_{k_1,...,k_r,m_1,...,m_r}\left(z_1,z_2,...,z_r\right)= \\
&=\mathbf{E}\left\{x\left(z_1\right)^{k_1}...x\left(z_r\right)^{k_r}x^*\left(z_1\right)^{m_1}...x^*\left(z_r\right)^{m_r}\right\},
\end{aligned}
\tag{1}
$$

where symbol «*» stands for complex conjugation, and $\mathbf{E}$ is an operator of probabilistic average.

A set of such polynomial moments in fact specifies the probability density and characteristic function Θ of a complex random vector $\mathbf{x}\in C^n$ formed by r values of random polynomial $x(z)$ in points $z_1,...,z_r\in C$:

$$
\begin{aligned}
&\Theta\left(p_1,...,p_r;z_1,...,z_r\right)= \\
&=1+\sum_{l=1}^{\infty}j^l\sum_{m_1,...,m_r=l}\frac{1}{m_1!...m_r!}\sum_{k_1...k_r=0}^{m_1,...,m_r}\binom{k_1}{m_1}\cdot...\cdot\binom{k_r}{m_r}\times \\
&\times P^x_{k_1,...,k_r,m_1,...,m_r}\left(z_1,z_2,...,z_r\right)\times \\
&\times p_1^{m_1-k_1}\left(p_1^*\right)^{k_1}...p_r^{m_r-k_r}\left(p_r^*\right)^{k_r},
\end{aligned}
\tag{2}
$$

where $p_i=\frac{1}{2}\left(\omega_i^{\mathrm{Re}}-j\omega_i^{\mathrm{Im}}\right)$, and $\binom{k}{m}$ is a binomial coefficient.

The probability density of complex coefficients of a random polynomial can be found by calculating the $2r$-dimensional Fourier inverse for the characteristic function (2).

If $y(z)=h(z)x(z)$ is a product of random polynomial $x(z)$ and nonrandom polynomial $h(z)$, then

$$
P_l^y\left(z_1,...,z_l\right)=P_l^x\left(z_1,...,z_l\right)\cdot h\left(z_1\right)\cdot...\cdot h\left(z_l\right). \tag{3}
$$

If $y(z)=x_1(z)x_2(z)$ is a product of independent random polynomials $x_1(z)$ and $x_2(z)$ (i.e. polynomials whose coefficient vectors $\mathbf{x}_1$ and $\mathbf{x}_2$ are independent), then

$$
P_l^y\left(z_1,...,z_l\right)=P_l^{x_1}\left(z_1,...,z_l\right)\cdot P_l^{x_2}\left(z_1,...,z_l\right). \tag{4}
$$

If $y(z)=x_1(z)+x_2(z)$ is a sum of independent random polynomials $x_1(z)$ and $x_2(z)$, then

$$
\begin{aligned}
&P_l^y\left(z_1,...,z_l\right)= \\
&=\sum_{r=1}^{l}\left(\sum_{\substack{0<i_1<...<i_r<l\\0<i_{r+1}<...<i_l<l}}P_r^{x_1}\left(z_{i_1},...,z_{i_r}\right)P_{l-r}^{x_2}\left(z_{i_{r+1}},...,z_{i_l}\right)\right).
\end{aligned}
\tag{5}
$$

This expression shows that addition of polynomial moments of independent random polynomials isn't commutative. But addition of polynomial cumulants has commutative property.

Let us define polynomial cumulant of order $(k+m)$, where $k=k_1+k_2+...+k_r$ and $m=m_1+m_2+...+m_r$, of random vector $\mathbf{x}\in C^n$ as a polynomial with r variables, belonging to ring $C\left[z_1,...,z_r\right]$:

$$
\begin{aligned}
&K^x_{k_1,...,k_r,m_1,...,m_r}\left(z_1,z_2,...,z_r\right)= \\
&=\mathrm{cum}\left\{x\left(z_1\right)^{k_1}...x\left(z_r\right)^{k_r}x^*\left(z_1\right)^{m_1}...x^*\left(z_r\right)^{m_r}\right\},
\end{aligned}
\tag{6}
$$

where cum is symbol of random value cumulant.

The relation between the characteristic function Θ of r values of a random polynomial $x(z)$ in points $z_1,...,z_r\in C$ and the set of polynomial cumulants can be described as follows:

$$
\begin{aligned}
&\ln\left(\Theta\left(p_1,...,p_r;z_1,...,z_r\right)\right)= \\
&=\sum_{l=1}^{\infty}j^l\sum_{m_1,...,m_r=l}\frac{1}{m_1!...m_r!}\sum_{k_1,...,k_r=0}^{m_1,...,m_r}\binom{k_1}{m_1}\cdot...\cdot\binom{k_r}{m_r}\times \\
&\times K^x_{k_1,...,k_r,m_1,...,m_r}\left(z_1,z_2,...,z_r\right)p_1^{m_1-k_1}\left(p_1^*\right)^{k_1}\cdot...\cdot p_r^{m_r-k_r}\left(p_r^*\right)^{k_r}.
\end{aligned}
\tag{7}
$$

For any given polynomial cumulant we can determine a set of points in space C^r for which the polynomial cumulant has zero value:

$$\Xi^x_{k_1,\dots,k_r,m_1,\dots,m_r} = \left\{ z \in C^r : K^x_{k_1,\dots,k_r,m_1,\dots,m_r}\left(z_1, z_2, \dots, z_r\right) = 0 \right\}. \tag{8}$$

Such points form a set of polynomial cumulant roots in ring $C\left[z_1,\dots,z_r\right]$ and an affine manifold in space C^r.

Let us see how polynomial cumulants and manifolds they generate, can help identify statistical relations between components of a random vector.

The l-th order zero-correlation manifold of r values of random polynomial $x\left(z\right)$ is described as follows:

$$\Xi^x_l = \begin{Bmatrix} z \in C^r : K^x_{k_1\dots k_r, m_1\dots m_r}\left(z_1,\dots,z_r\right) = 0, \\ k_1 + \dots + k_r + m_1 + \dots + m_r = l, \\ k_1 + m_1 > 1, \dots, k_r + m_r > 1 \end{Bmatrix}. \tag{9}$$

Let $\mathbf{x} \in R^n$ be a random vector with probability density $f_x\left(x_1,\dots,x_n\right)$ in R^n. Let $x\left(z\right) \in C\left[z\right]$ be a random polynomial of degree $n-1$ with vector of coefficients $\mathbf{x} \in R^n$.

For any cumulant $K^x_{k_1,\dots,k_r,m_1,\dots,m_r}\left(z_1, z_2, \dots, z_r\right)$ we can identify a set of points in space C^r for which the polynomial cumulant has a given value $t \in C$:

$$\Xi^x_{k_1,\dots,k_r,m_1,\dots,m_r}\left(t\right) = \left\{ z \in C^r : K^x_{k_1,\dots,k_r,m_1,\dots,m_r}\left(z_1, z_2, \dots, z_r\right) = t \right\}. \tag{10}$$

We specifically can find all possible values $z_1 \neq z_2$ for which $x\left(z_1\right)$ and $x\left(z_2\right)$ have a given value of the first correlation function. This can be done by solving the following system of polynomial equations:

$$\Xi^x_{1,1,0,0}\left(t\right) = \left\{ z \in C^r : K^x_{1,1,0,0}\left(z_1, z_2\right) = t,\ t \in C \right\}. \tag{11}$$

Affine manifold $\Xi^x_{1,1,0,0}\left(t\right)$ in C^2 is called a first-order manifold with a given correlation t of random polynomial $x\left(z\right)$.

Let us assume, for instance, that $x\left(z\right) \in C[z]$ is a random polynomial of degree $n-1$ specified by a Gaussian random vector with zero probabilistic average, independent components and variance of components σ^2. Then the manifold with a given correlation of the random polynomial values $x(z_1)$ and $x(z_2)$ looks like this:

$$\Xi^x_{1,1,0,0}\left(t\right) = \left\{ \left(z_1, z_2\right) \in C^2 : z_1 z_2 = \alpha_i\left(t\right),\ \ i = \overline{1, n-1} \right\}, \tag{12}$$

where $\alpha_1,\dots,\alpha_{n-1}$ are the polynomial roots:

$$P\left(x\right) = (1 - t/\sigma^2) + x + x^2 + \dots + x^{n-1}. \tag{13}$$

Let $x_1\left(z\right)$ and $x_2\left(z\right)$ be independent random polynomials, and let $\Xi^{x_1}_{k_1,\dots,k_r,m_1,\dots,m_r}\left(t_1\right)$ and $\Xi^{x_2}_{k_1,\dots,k_r,m_1,\dots,m_r}\left(t_2\right)$ be their respective manifolds with a given correlation. Then zero-correlation manifolds obtained by multiplying and adding the respective polynomials, are described as follows:

$$\Xi^{x_1 x_2}_{k_1,\dots,k_r,m_1,\dots,m_r}\left(0\right) = \Xi^{x_1}_{k_1,\dots,k_r,m_1,\dots,m_r}\left(0\right) \bigcup \Xi^{x_1}_{k_1,\dots,k_r,m_1,\dots,m_r}\left(0\right), \tag{14}$$

$$\Xi^{x_1 + x_2}_{k_1,\dots,k_r,m_1,\dots,m_r}\left(0\right) = \Xi^{x_1}_{k_1,\dots,k_r,m_1,\dots,m_r}\left(t\right) \bigcap \Xi^{x_2}_{k_1,\dots,k_r,m_1,\dots,m_r}\left(-t\right). \tag{15}$$

Let $x_1\left(z\right), x_2\left(z\right), \dots, x_n\left(z\right)$ be a set of independent random polynomials, and let

$\Xi^{x_1}_{k_1,\dots,k_r,m_1,\dots,m_r}\left(t_1\right), \dots, \Xi^{x_n}_{k_1,\dots,k_r,m_1,\dots,m_r}\left(t_n\right)$ be their respective manifolds with a given correlation. Then

$$\Xi^{x_1 \dots x_n}_{k_1,\dots,k_r,m_1,\dots,m_r}\left(0\right) = \bigcup_{i=1}^{n} \Xi^{x_i}_{k_1,\dots,k_r,m_1,\dots,m_r}\left(0\right), \tag{16}$$

$$\Xi^{x_1 \dots x_n}_{k_1,\dots,k_r,m_1,\dots,m_r}\left(t_1, t_2, \dots, t_n\right) = \bigcap_{i=1}^{n} \Xi^{x_i}_{k_1,\dots,k_r,m_1,\dots,m_r}\left(t_i\right), \tag{17}$$

$$\Xi^{x_1 + x_2 + \dots + x_n}_{k_1,\dots,k_r,m_1,\dots,m_r}\left(t_1 + t_2 + \dots + t_n\right) = \bigcap_{i=1}^{n} \Xi^{x_i}_{k_1,\dots,k_r,m_1,\dots,m_r}\left(t_i\right). \tag{18}$$

In (17) and (18) $t_i \neq 0$.

A manifold $\Xi \subset C^r$ is called irreducible when it can be presented as $\Xi = \Xi_1 \bigcup \Xi_2$, where Ξ_1 and Ξ_2 are affine manifolds, if and only if $\Xi_1 = \Xi$ or $\Xi_2 = \Xi$.

If $\Xi \subset C^r$ is an affine manifold, then there is only one way to represent it as follows:

$$\Xi = \bigcup_{i=1}^{n} \Xi_i, \tag{19}$$

where every Ξ_i is irreducible and $\Xi_i \not\subset \Xi_j$, $i \neq j$.

Thus any affine manifold can be obtained or represented as a finite union of irreducible manifolds (so states the corollary of Hilbert's theorem on the finite nature of an ideal) [9].

Now we let us look at some simple cases illustrating visual features of reducible and irreducible manifolds.

Example 1

Let $x\left(z\right) \in C\left[z\right]$ be a random polynomial of degree $n-1$ specified by random vector of coefficients $x \in R^n$ with zero probabilistic average, independent components and variance of components σ^2. Then the zero correlation manifold of a random polynomial can be factored to a union of $n-1$ irreducible manifolds:

$$\Xi^x = \bigcup_{i=1}^{n-1} \Xi_i, \tag{20}$$

where $\Xi_i = \left\{ \left(z_1, z_2\right) \in C^2 : z_1 z_2 = \alpha_i \right\}$ and $\alpha_i, i = \overline{1, n-1}$ are roots of a reducible polynomial

$P\left(x\right) = 1 + x + x^2 + \ldots + x^{n-1}$.

Let us assume that

$z_1 = x + yj, z_2 = u + vj$ and $\alpha_i = a_i + b_i j, i = \overline{1, n-1}$;

then each equation $z_1 z_2 = \alpha_i$ corresponds to the following system of equations:

$$\begin{cases} x = \dfrac{a_i u + b_i v}{u^2 + v^2}, \\ y = \dfrac{b_i u - a_i v}{u^2 + v^2}, \end{cases} \tag{21}$$

which specifies an irreducible four-dimensional manifold $\Xi_i \in R^4$.

The parametric form of (21) looks as follows:

$$\begin{cases} x = \dfrac{a_i h + b_i t}{h^2 + t^2}, \\ y = \dfrac{b_i h - a_i t}{h^2 + t^2}, \\ u = h, \\ v = t. \end{cases} \tag{22}$$

With $t = v$ viewed as the so-called time parameter, a four-dimensional zero-correlation manifold Ξ^x can be looked at as a moving three-dimensional manifold. So, in R^3 we have

$$\begin{cases} x = \dfrac{a_i h + b_i t}{h^2 + t^2}, \\ y = \dfrac{b_i h - a_i t}{h^2 + t^2}, \\ u = h, \end{cases} \tag{23}$$

where t is a time parameter.

It should be noted that there is only one spatial parameter in (23), h. This implies that irreducible components Ξ_i of correlated manifold Ξ^x are spatial curves. To get a better understanding of how they move, the curves are shown as surfaces which they form as they move in R^3, for $t \in [0,20]$.

In Fig. (**1**) illustrating the case where $n = 6$ we can see five various irreducible submanifolds.

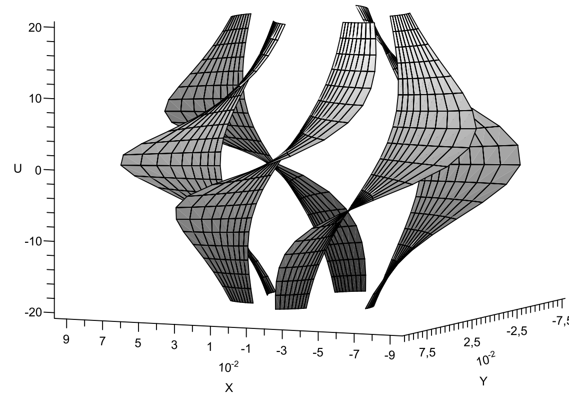

Fig. (1). The reducible zero-correlation manifold (23) as a parametric function of spatial parameter $h \in [10,20]$ and time parameter $t \in [0,20]$.

Example 2

Let us consider a random polynomial of the first degree $x\left(z\right) \in C\left[z\right]$ with nonzero correlation of coefficients. In this case zero-correlation manifold Ξ^x is generated by irreducible polynomial

$$K^x_{1,1,0,0}\left(z_1, z_2\right) = r_{00} + r_{01}\left(z_1 + z_2\right) + r_{11} z_1 z_2, \tag{24}$$

and therefore is irreducible too.

Fig. (**2**) shows a zero-correlation manifold obtained for random polynomial $x(z)$ for the case where variance of components $r_{00} = r_{11} = 1$, correlation $r_{01} = \dfrac{1}{2}$ and variation of time parameter $t \in [0,20]$.

Assuming that $z_1 = x + yj$ and $z_2 = u + vj$, we can obtain from equation $r_{00} + r_{01}\left(z_1 + z_2\right) + r_{11} z_1 z_2 = 0$ the following system of equalities:

$$\begin{cases} x = -\dfrac{1}{2} \cdot \dfrac{u^2 + 2{,}5u + 1 + v^2}{(u + 0{,}5)^2 + v^2}, \\ y = \dfrac{3}{4} \cdot \dfrac{v}{(u + 0{,}5)^2 + v^2}, \end{cases} \tag{25}$$

or, in a parametric form:

$$\begin{cases} x = -\dfrac{1}{2} \cdot \dfrac{h^2 + 2{,}5h + 1 + t^2}{(h + 0{,}5)^2 + t^2}, \\ y = \dfrac{3}{4} \cdot \dfrac{t}{(h + 0{,}5)^2 + t^2}, \\ u = h, \\ v = t. \end{cases} \tag{26}$$

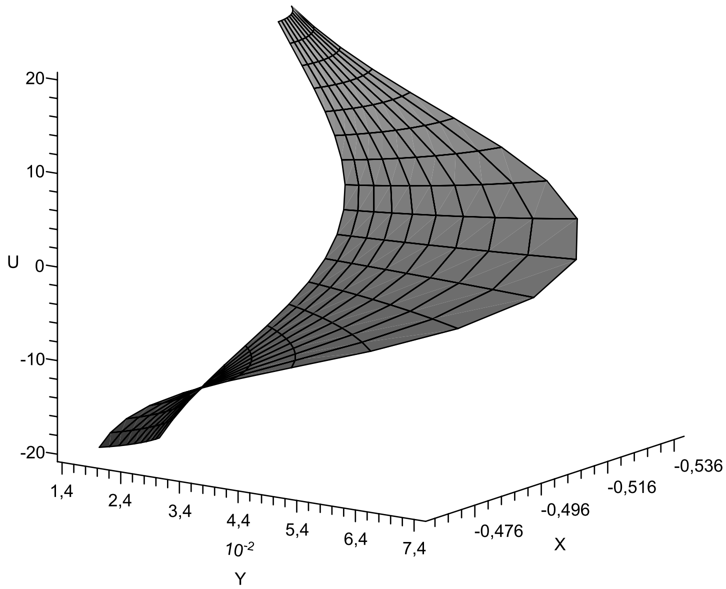

Fig. (2). The irreducible zero-correlation manifold (27) as a parametric function of spatial parameter $h \in [10,20]$ and time parameter $t \in [0,20]$.

If t is a time parameter, then zero-correlation manifold Ξ^x is given by:

$$\begin{cases} x = -\frac{1}{2} \cdot \frac{h^2 + 2,5h + 1 + t^2}{(h+0,5)^2 + t^2}, \\ y = \frac{3}{4} \cdot \frac{t}{(h+0,5)^2 + t^2}, \\ u = h, \end{cases} \tag{27}$$

where h is a spatial parameter.

In the following section we are going to look at how polynomial statistics can be applied for solving the blind identification problem.

III. IMPULSE RESPONSE CHANNEL IDENTIFICATION BY GIVEN CORRELATION MANIFOLDS

Now we consider approaches to solving the problem of blind identification of sequential systems which transmit discrete messages with a passive pause (i.e. each data burst is separated from the next one by a pause). In this case, the system is identifiable blindly because its input signal is a non-stationary random process.

For a passive pause system a channel model can be described by a linear combination of polynomials of positive degree:

$$y(z) = h(z)x(z) + v(z). \tag{28}$$

In this expression $y(z), h(z), x(z), v(z) \in C[z]$ are polynomials representing respectively the discrete output signal, the final discrete pulse response of the channel, the input information sequence and noise. We regard random polynomials as complex random fields defined on the complex plane. In this case the moment and cumulant functions of these random fields can be defined as multivariable polynomials [13].

The equation linking the polynomial cumulants for the input and output of an identifiable system using a passive pause (28) can be written down as follows:

$$\begin{aligned} K^y_{k,m}(z_1,...,z_r) &= h(z_1)^{k_1} \dots h(z_r)^{k_r} \times \\ &\times h^*(z_1)^{m_1} \dots h^*(z_r)^{m_r} K^x_{k,m}(z_1,...,z_r) + \\ &+ K^v_{k,m}(z_1,...,z_r). \end{aligned} \tag{29}$$

In case where the information sequence statistics is not known for sure we can use zero-correlation manifold structure for blind identification. If the noise statistics is known, then (28) can be re-written as follows:

$$\Xi^{y-v}_{k,m}(0) = \Xi^h_{k,m}(0) \bigcup \Xi^x_{k,m}(0), \tag{30}$$

where $$\Xi^x_{k,m}(t) = \left\{ K^x_{k,m}(z_1,...,z_r) = t,\ t \in C \right\}, \tag{31}$$

$$\Xi^h_{k,m}(t) = \left\{ K^h_{k,m}(z_1,...,z_r) = t,\ t \in C \right\}, \tag{32}$$

$$\Xi^{y-v}_{k,m}(t) = \left\{ K^y_{k,m}(z_1,...,z_r) - K^v_{k,m}(z_1,...,z_r) = t,\ t \in C \right\}. \tag{33}$$

As it was stated above, any manifold can be represented as a finite union of irreducible manifolds, and for each manifold such representation is unique. If $\Xi^h_{k,m}(0) \not\subset \Xi^x_{k,m}(0)$, then representation (30) is unique. Therefore, manifold $\Xi^h_{k,m}(0)$ fully describes the channel impulse response and can be found by presenting manifold $\Xi^{y-v}_{k,m}(0)$ as a union of irreducible manifolds. In this case no prior knowledge of information sequence moments is required. In a complex field, however, presenting a manifold as a union of irreducible manifolds can be a problem. Still, there is one property that distinguishes manifolds generated by channel impulse response from those generated by information sequence that is the dimension of manifolds. We are going to take advantage of this property.

Zero-correlation manifold $\Xi^{h}_{k,m}\left(0\right)$ is generated by a finite point set in C (zero-dimensional manifold in $C = R^2$) and appears to be a union of complex hyperplanes in C^r. Manifold $\Xi^{x}_{k,m}\left(0\right)$ usually has dimension 1 in C. In case of independent identically distributed samples of the information sequence this manifold is a union of complex hypersurfaces in C^r. We can, therefore, distinguish between unknown manifolds as long as they have different dimensions; the difference becomes obvious as we make various cross-cuts. For $r = 2$ the blind identification algorithm (A1) consists of the following sequence of operations:

1. Estimate polynomial covariance $\hat{P}^{y}_{2,0}\left(z_1, z_2\right)$ on the basis of M representations of the output signal.
2. Calculate vectors with one variable polynomial roots, $\mathbf{r}_1 = \mathrm{roots}\left(\hat{P}^{y}_{2,0}\left(z_1, z_2^1\right)\right)$ and $\mathbf{r}_2 = \mathrm{roots}\left(\hat{P}^{y}_{2,0}\left(z_1, z_2^2\right)\right)$, $z_2^1 \neq z_2^2$.
3. Using criterion $\left\|\mathbf{r}_1 - \mathbf{r}_2\right\| \le \varepsilon\left(\sigma^2\right)$, form vector $\mathbf{r}_h$ which contains L nearest roots in plane C.
4. Find the estimate $\hat{\mathbf{h}} = \mathrm{roots}^{-1}\left(\mathbf{r}_h\right)$.

Generally, for $r > 2$ the algorithm applied to discern manifolds remains basically the same. The projection $C^r \to C$ of impulse response generated manifold $\Xi^{h}_{k,m}\left(0\right)$ on the first coordinate axis has zero dimension in C, whereas the projection of manifold $\Xi^{x}_{k,m}\left(0\right)$ generated by information sequence has dimension no less than 1 in C.

Zero-correlation manifolds $\Xi^{y-v}_{k,m}\left(0\right)$ obtained by using this algorithm for the reducible (23) and irreducible (27) input sequence polynomials $x(z) \in C[z]$ from examples 1 and 2 are shown in Figs. (**3** and **4**), respectively. The estimations are based on the assumption that the channel impulse response polynomial is a first-degree determinate polynomial $h(z) = h_0 + h_1 z$ with complex coefficients $h_0 = 1 - j$ and $h_1 = 2 + 3j$, which has a single root, $-\dfrac{h_0}{h_1} = \dfrac{1}{13} + \dfrac{5}{13}j$.

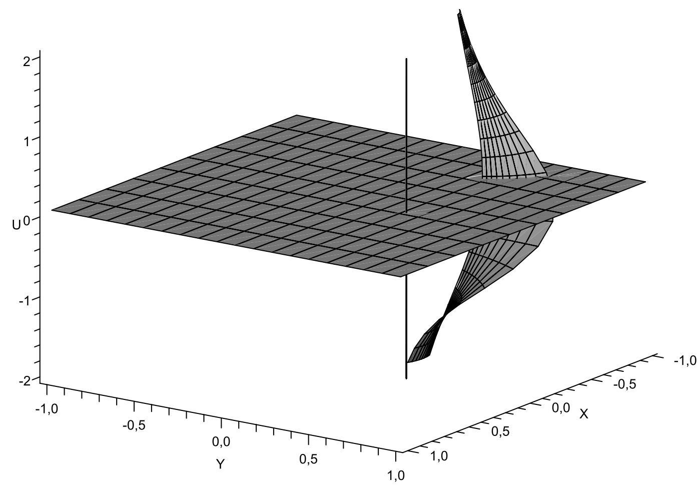

Fig. (3). The manifold of observed signal $y(z) \in C[z]$ without noise for reducible zero-correlation manifold (23) of input signal $x(z) \in C[z]$.

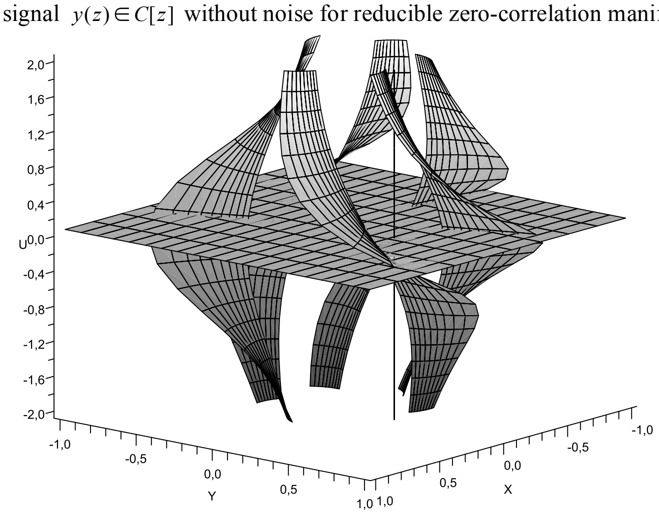

Fig. (4). The manifold of observed signal $y(z) \in C[z]$ without noise for irreducible zero-correlation manifold (27) of input signal $x(z) \in C[z]$.

Thus, the zero-correlation manifold generated by $h(z)$ looks like this:

$$\Xi^h_{1,1,0,0}\left(0\right)=\left\{(z_1,z_2)\in C:K^h_{1,1,0,0}\left(z_1,z_2\right)=0\right\}= \\ =\left\{(z_1,z_2)\in C:h(z_1)h(z_2)=0\right\}, \tag{34}$$

$$\Xi^h_{1,1,0,0}\left(0\right)=\Xi^h_1\cup\Xi^h_2= \\ =\left\{(z_1,z_2)\in C:z_1=-\frac{h_0}{h_1}\right\}\cup\left\{(z_1,z_2)\in C:z_2=-\frac{h_0}{h_1}\right\}. \tag{35}$$

If we look at manifold projections onto complex plane C, we will see a number of points in the case of sub-manifold $\Xi^h_{1,1,0,0}\left(0\right)\in C^2$ generated by channel pulse response, and a number of curves in the case of manifold $\Xi^x_{1,1,0,0}\left(0\right)\in C^2$ of information sequence. These objects are clearly seen in Figs. (**3** and **4**) at the intersection of complex plane C and manifolds. It is worth noticing that, whatever the secant plane is, a vertical straight line Ξ^h_2 always projects onto a point (i.e. a zero-dimensional manifold in C), whereas a surface $\Xi^x\left(0\right)$ generated by input signal $x(z)\in C[z]$ always projects into a number of curves (a first- dimensional manifolds in C).

In case of prior knowledge of the input signal statistics a blind identification algorithm can be developed using directly a given correlation manifold structure of a random polynomial. Let $x\left(z\right)\in C\left[z\right]$ be a random polynomial of degree $n-1$ specified by random Gaussian vector $\mathbf{x}\in R^n$ with zero probabilistic average, independent components and variance of components σ^2. Then a manifold with a given correlation of random polynomial values looks as follows:

$$\Xi^x_{1,0,1,0}\left(t\right)=\left\{\left(z_1,z_2\right)\in C^2:z_1z_2^{\ *}=\alpha_i\left(t\right),\ \ i=\overline{1,n-1}\right\}, \tag{36}$$

where $\alpha_1,\ldots,\alpha_{n-1}$ are roots of polynomial $P\left(x\right)=(1-t/\sigma^2)+x+x^2+\ldots+x^{n-1}$.

Let us now look at the case where points are chosen so that pair correlations $t_{i,j}$ of components are nonvanishing and not equal to each other; in other words, they may belong to different manifolds with a given correlation.

Let us assume that $\alpha_1,\ldots,\alpha_{n-1}$ are roots of polynomial $P\left(x\right)$. It is possible to demonstrate that, as long as $t_{i,j}\neq 0$, no pair of these roots belongs to $\Xi^x_{1,0,1,0}\left(0\right)$. It means that the second mixed cumulant has the following form:

$$\frac{K^y_{1,0,1,0}\left(\alpha_i,\alpha_j\right)}{t_{i,j}}=h\left(\alpha_i\right)h^*\left(\alpha_j\right), \\ i=\overline{1,n-1},\quad j=\overline{1,n-1},\quad t_{i,j}\neq 0. \tag{37}$$

Thus we can construct a linear mapping of vector $\mathbf{x}\in C^n$ into vector $\mathbf{y}\in C^{n-1}$, such that its first and second covariance matrices have nonzero off-diagonal entries. It means that channel estimation, in fact, boils down to finding the eigenvector with the maximum eigenvalue [12]. The blind identification algorithm (A2), therefore, consists of the following sequence of operations:

1. Transformation of pair correlations of the output signal

$$\vec{s}_k=\mathbf{V}_n\left(\alpha_1,\ldots,\alpha_{n-1}\right)\vec{y}_k=\begin{pmatrix}1&\alpha_1&\cdots&\alpha_1^{n-1}\\ \vdots&\vdots&&\vdots\\ 1&\alpha_{n-1}&\cdots&\alpha_{n-1}^{n-1}\end{pmatrix}\begin{pmatrix}y_0\\ \vdots\\ y_{n-1}\end{pmatrix}_k, \tag{38}$$

where $\mathbf{V}_{n-1}\left(\alpha_1,\ldots,\alpha_{n-1}\right)$ is a Vandermond $\left(n-1\right)\times n$ matrix, and $\vec{y}_k$ is a k-th vector of the output signal samples.

2. Estimation of sample covariation matrix

$$\hat{\mathbf{r}}=\frac{1}{M}\sum_{k=1}^{M}\vec{s}_k\vec{s}_k^{\ *}. \tag{39}$$

3. Calculation of the matrix eigenvector $\mathbf{R}=\left(\hat{r}_{i,j}/t_{i,j}\right)$,

$$\begin{pmatrix}h\left(\alpha_1\right)\\ \vdots\\ h\left(\alpha_{n-1}\right)\end{pmatrix}=\arg\left(\max_{\vec{x}^*\vec{x}=1}\left(\vec{x}^*\mathbf{R}\vec{x}\right)\right). \tag{40}$$

4. Calculation of channel pulse response

$$\begin{pmatrix}h_0\\ \vdots\\ h_{L-1}\end{pmatrix}=\mathbf{V}_L^{\#}\left(\alpha_1,\ldots,\alpha_{n-1}\right)\begin{pmatrix}h\left(\alpha_1\right)\\ \vdots\\ h\left(\alpha_{n-1}\right)\end{pmatrix}, \tag{41}$$

where symbol « # » stands for Moore-Penrose inversion.

IV. RESULTS OF MATHEMATICAL SIMULATION

To evaluate the efficiency of the proposed solution we will compare it with a well-known moment method based on using cumulant spectrums [9]. It is shown in [13] that blind identification of a non-stationary input cannel requires solving the following algebraic equation for second-order spectral moments:

$$\dot{F}_{yy}\left(m,n\right)=\dot{H}\left(m\right)H^*\left(n\right)\dot{F}_{xx}\left(m-n\right)+\dot{F}_{vv}\left(m-n\right), \tag{42}$$

$$\dot{F}_{xx}\left(m\right)=\sum_{k=0}^{N-1}g_k^2\exp\left(-j\frac{2\pi km}{N}\right), \tag{43}$$

$$\dot{F}_{vv}\left(m\right)=N_0\delta\left(m\right), \tag{44}$$

where $\dot{H}\left(m\right)$ is a channel transfer function, $n=\overline{0,N-1}$ and $m=\overline{0,N-1}$.

Second order spectral moments in (42)-(44) are described by the following expression:

$$\dot{F}_{xy}\left(m,n\right)=\mathbf{E}\left\{\sum_{k=0}^{N-1}x(k)\exp\left(-j\frac{2\pi km}{N}\right)\sum_{k=0}^{N-1}y^{*}(k)\exp\left(j\frac{2\pi kn}{N}\right)\right\}. \tag{45}$$

This expression implies that spectral moments of information sequence and noise are known, whereas the spectral moment of channel output sample sequence is estimated on the basis real observations. Algorithms that solve (42) for an unknown transfer function of the channel become easy to develop if we assume that equation (42) is true for estimate $\hat{F}_{yy}\left(n,m\right)$. The algorithm (A3) which uses spectral factorization minimizes the root-mean-square error of random solutions of (42) as long as the energy of transfer function is normalized and $\left|\dot{F}_{xx}\left(m\right)\right|\neq 0$,

$$\hat{H}\left(m\right)=\arg\min_{H}\left(\sum_{m=0}^{N-1}\sum_{n=0}^{N-1}\left|\frac{\hat{F}_{yy}\left(m,n\right)-N_0\delta\left(m-n\right)}{\left|\dot{F}_{xx}\left(m-n\right)\right|^{2}}\times\right.\right. \tag{46}$$

$$\left.\left.\times F_{xx}^{*}\left(m-n\right)-\dot{H}\left(m\right)H^{*}\left(n\right)\right|^{2}\right).$$

It is known that in this case the solution is the eigenvector corresponding to the maximum eigenvalue of a Hermitian matrix.

Simulation results for algorithm (A3) are presented in Fig. (**5**). The relative error in this case is given by the formula $Q=\mathbf{E}\left\{\left\|\vec{h}-\hat{\vec{h}}\right\|/\left\|\vec{h}\right\|\right\}$. The pulse response is $\vec{h}=(0{,}7;\ 1{,}0;\ 0{,}7)$ for all experiments. Fig. (**6**) shows the mathematical simulation results for blind channel identification algorithm (A1), in which the channel is identified by two cross-cuts of zero-correlation manifold $V_{2,0}^{y-v}\left(0\right)\subset C^2$. In this case the two cutting planes are $\left\{z_2^1=1\right\}\subset C^2$ and $\left\{z_2^2=0{,}9\right\}\subset C^2$.

In comparison with algorithm (A3) algorithm (A1) is characterized by lower noise immunity for small values of noise-to-signal ratio. For a fixed sample noise immunity tends to zero. One important advantage of algorithm (A1) is the absence of the need to know information sequence statistics. Another important advantage is high rate of convergence. Thus, even for high noise-to-signal ratio (A1) provides acceptable error after just a few runs (N=3 to 5).

The simulation results for algorithm (A2) are presented in Fig. (**7**). The noise immunity of this algorithm is higher than that of algorithm (A3), whereas their rates of convergence are approximately the same. The use of nonzero correlation transformation provides for high noise immunity. The algorithm is, therefore, characterized by good matrix conditioning, unlike the algorithm based on spectral factorization where the condition $\left|\dot{F}_{xx}\left(m\right)\right|\neq 0$ is not met. The above-mentioned algorithms have approximately the same level of computational complexity.

CONCLUSION

The use of polynomial representations of random vectors resulted in a number of new blind channel identification algorithms based on methods of commutative algebra and al-

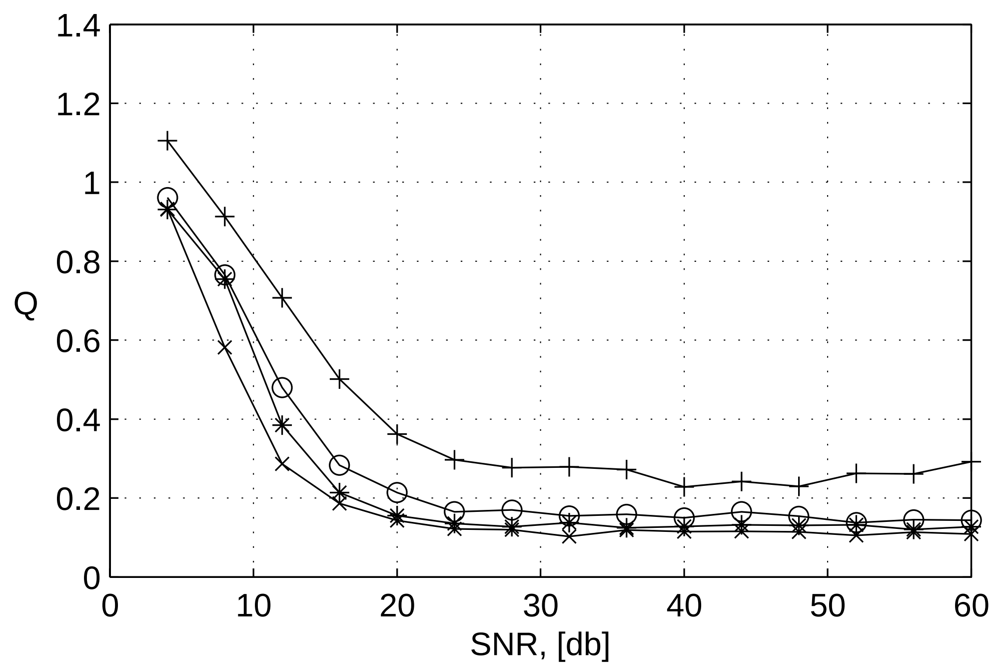

Fig. (5). Identification relative error Q of algorithm (A3) as a function of noise-to-signal ratio NSR, when a number of realizations N: N=20 («+»), N=40 («o»), N=60 («*»), N=80 («×»).

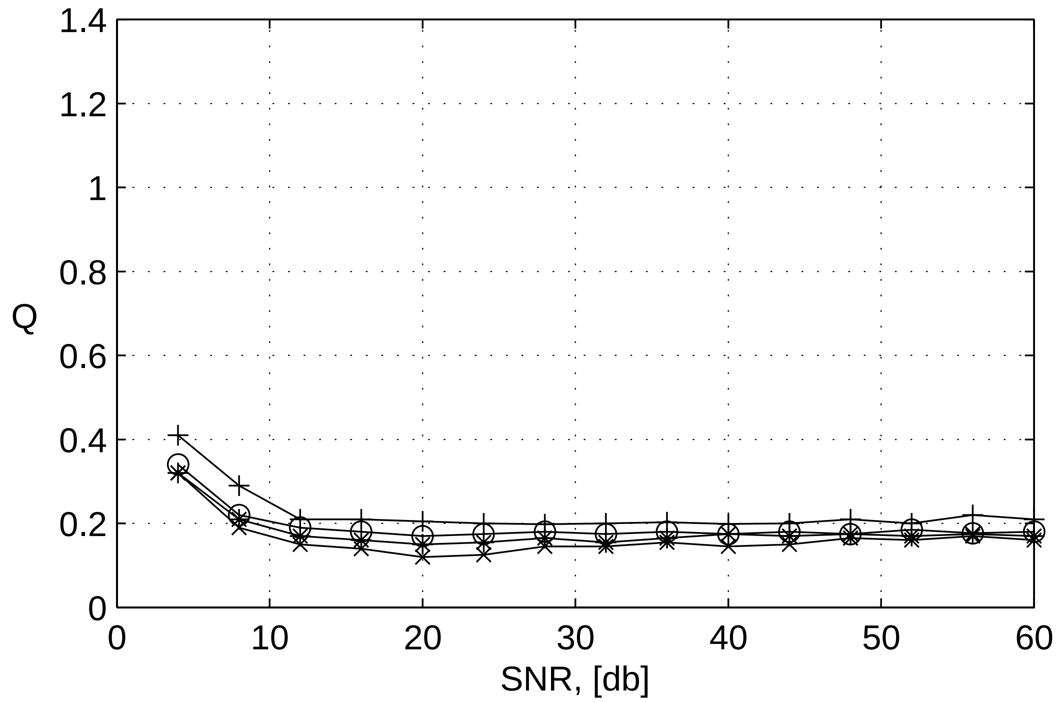

Fig. (6). Identification relative error Q of algorithm (A1) as a function of noise-to-signal ratio NSR, when a number of realizations N=20 and $\varepsilon = 0,01$ («+»), $\varepsilon = 0,03$ («o»), $\varepsilon = 0,05$ («*»), $\varepsilon = 0,07$ («×»).

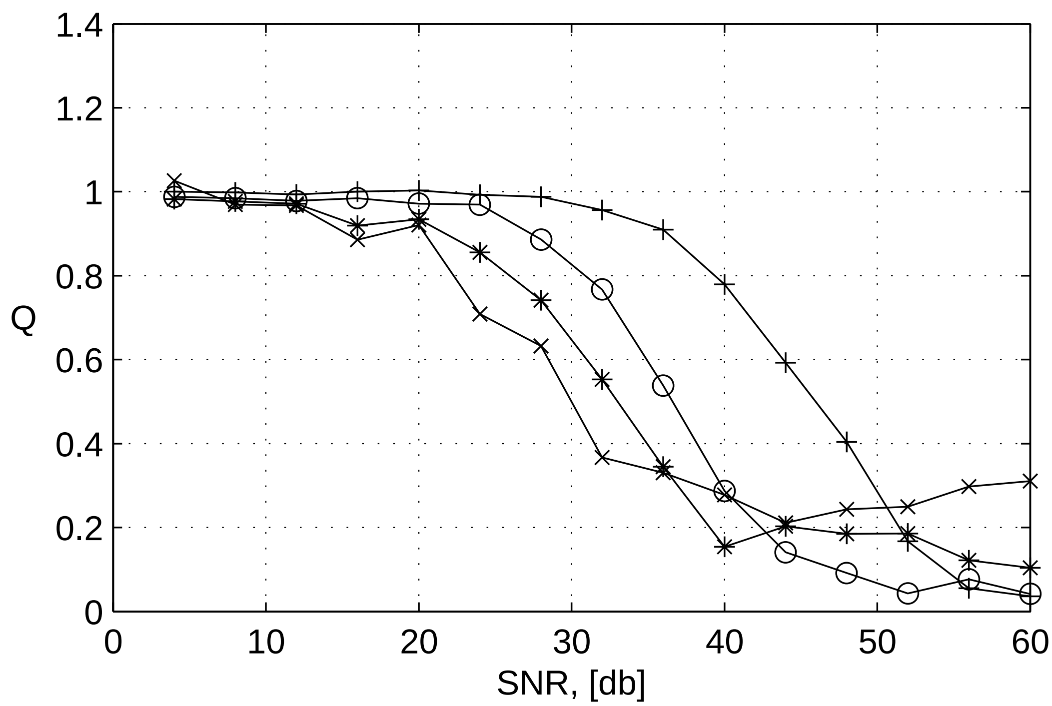

Fig. (7). Identification relative error Q of algorithm (A2) as a function of noise-to-signal ratio NSR, when $t = 100$ and a number of realizations N=20 («+»), N=40 («o»), N=60 («*»), N=80 («×»).

gebraic geometry. This paper demonstrates that manifolds generated by polynomial cumulants have a number of unique properties. For example, zero-correlation manifolds generated by random sequences transmitted through a determinate channel can be identified based on the number of dimensions they have, which means that blind channel identification is possible even in the absence of prior knowledge of input statistics.

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